TRINITY COLLEGE

Year 12 Mathematics Specialist Units 3, 4
Test 1   2020

Section 1   Calculator Free
Complex Numbers and Functions

STUDENT'S NAME   _____

DATE: Wednesday 4 March   **TIME:** 28 minutes   **MARKS**: 29

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.
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1.   (6 marks)

(a)   Express $5\, cis\, \dfrac{5\pi}{6}$ in the form $z = a + bi$ [3]

(b)   Express $\dfrac{\overline{2-i}}{(1+i)^2}$ in the form $z = a + bi$ [3]

2.   (5 marks)

Solve $z^4 + 8i = 0$. Answers may be given in polar form.

3.   (7 marks)

Consider the expression $z^4 + 3z^3 - 3z^2 + 3z - 4$
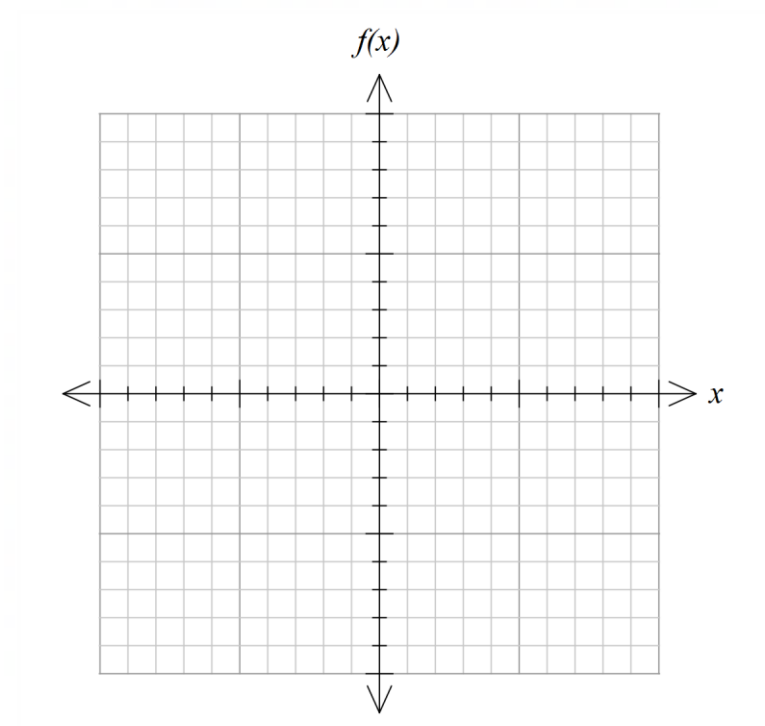
(a)   Show that $z - i$ is a factor of the above expression.   [2]

(b)   State another factor for the above expression.   [1]

(c)   Hence, or otherwise, solve $z^4 + 3z^3 - 3z^2 + 3z - 4 = 0$ [4]

4.   (5 marks)

Sketch the function $f(x) = \dfrac{3x - x^2}{x - 2}$, showing all intercepts, poles and asymptotes. It is not necessary to identify any stationary points.

5.   (6 marks)

Given $f(x) = \dfrac{3}{x^2 - 3}$ and $g(x) = \sqrt{x^2 - 1}$

(a)   By considering the restricted domain $\{x : x \in \mathbb{R}, x \geq 0, x \neq \sqrt{3}\}$, determine $f^{-1}(x)$ and state the restricted range of $f^{-1}(x)$.   [3]

(b)   Determine an expression for $f \circ g(x)$ and state the domain of $f \circ g(x)$.   [3]

Year 12 Mathematics Specialist Units 3, 4
Test 1   2020

Section 2   Calculator Assumed
Complex Numbers and Functions

STUDENT'S NAME   _____

DATE: Wednesday 4 March   **TIME:** 22 minutes   **MARKS**: 22

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)
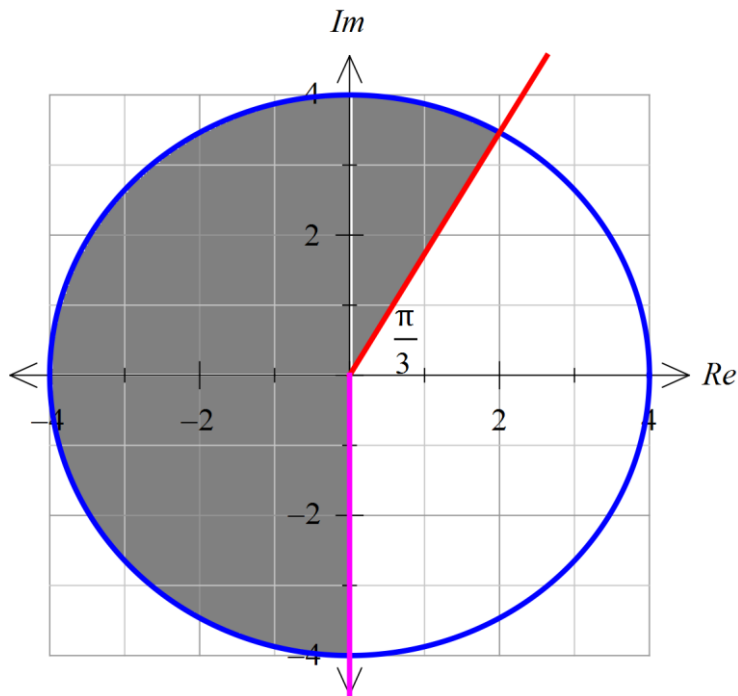
Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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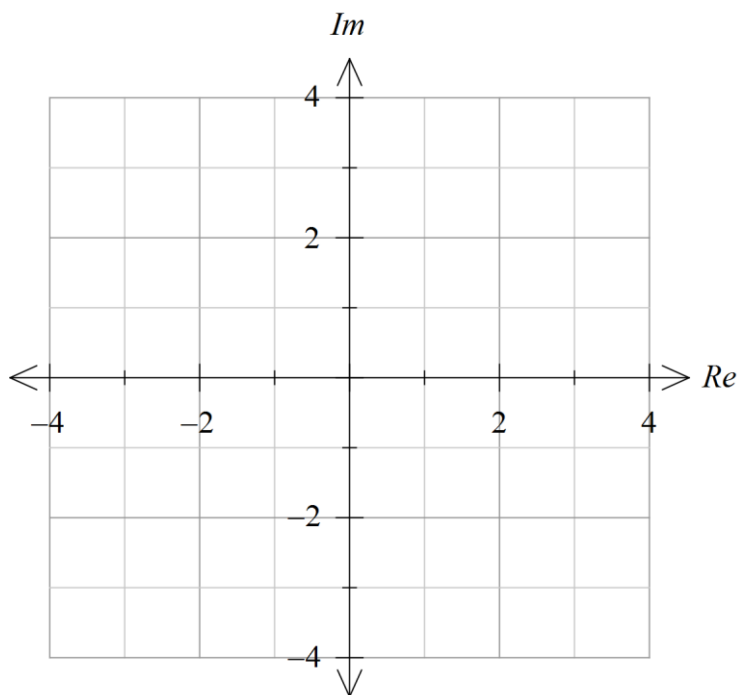*This page has been left blank intentionally*

6.   (11 marks)

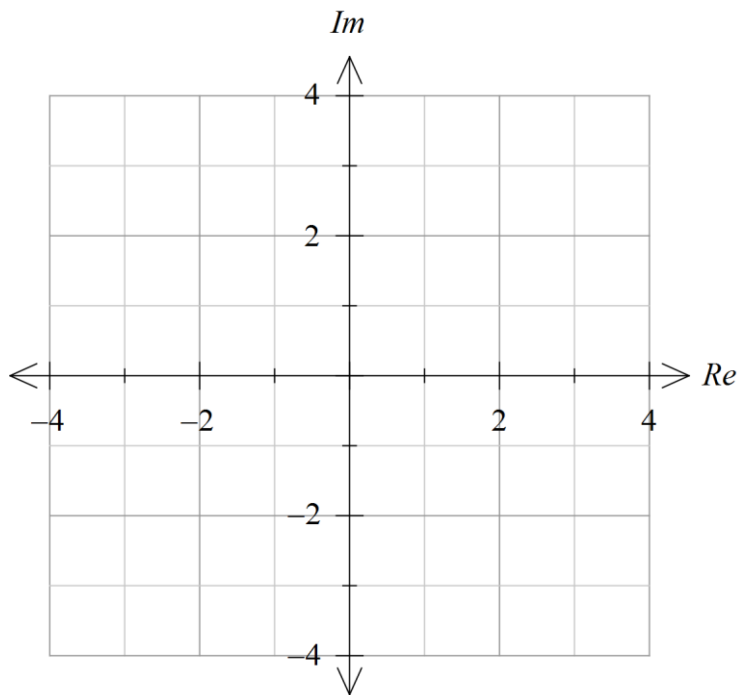   (a)   Describe fully the shaded region below   [3]

    (b)   Sketch the set of locus $\left\{ z : z + \bar{z} = 4 \right\}$ [3]
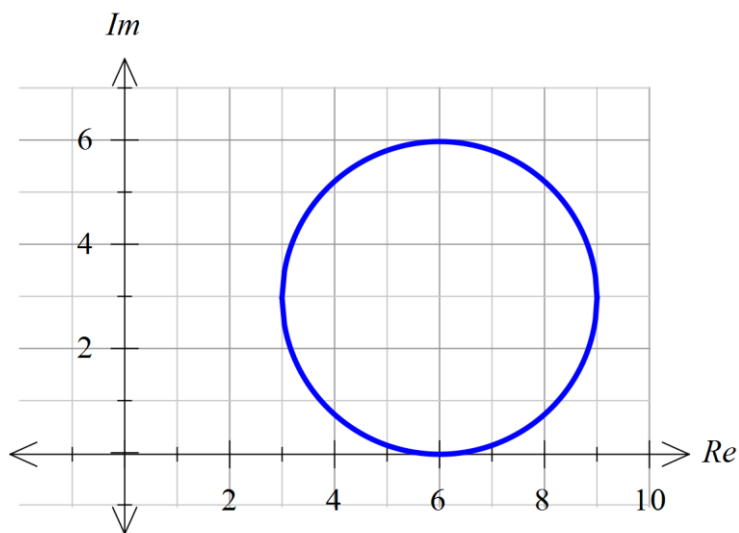
(c)   Sketch the set of locus $\left\{ z : \left| Arg(z-1) \right| \leq \dfrac{\pi}{4} \right\}$ [3]

(d)   The sketch of the locus of a complex number $\left\{ z : \left| z - 6 - 3i \right| = 3 \right\}$ is given below:

Determine the maximum value for $\left| z \right|$ as an exact value.   [2]
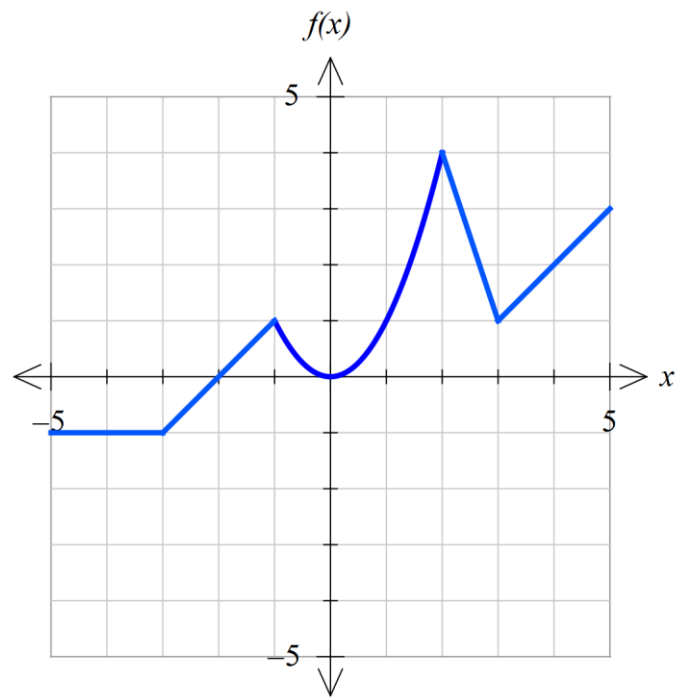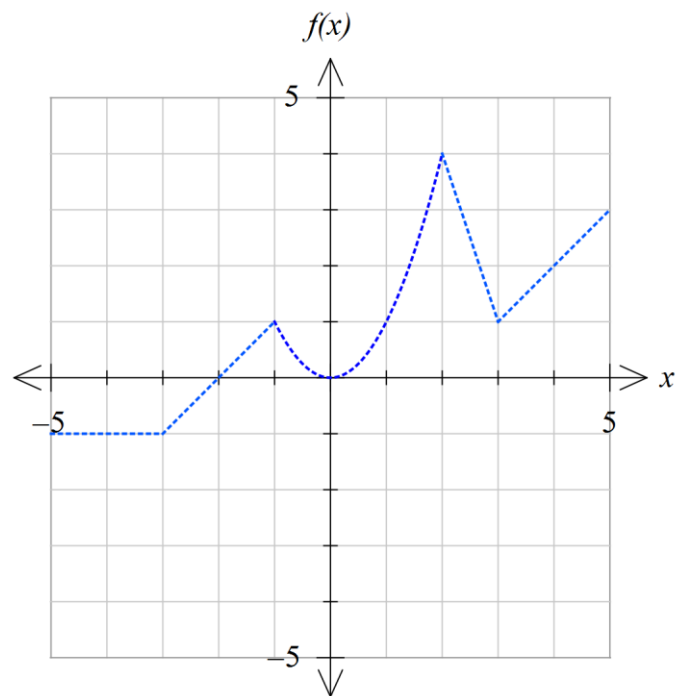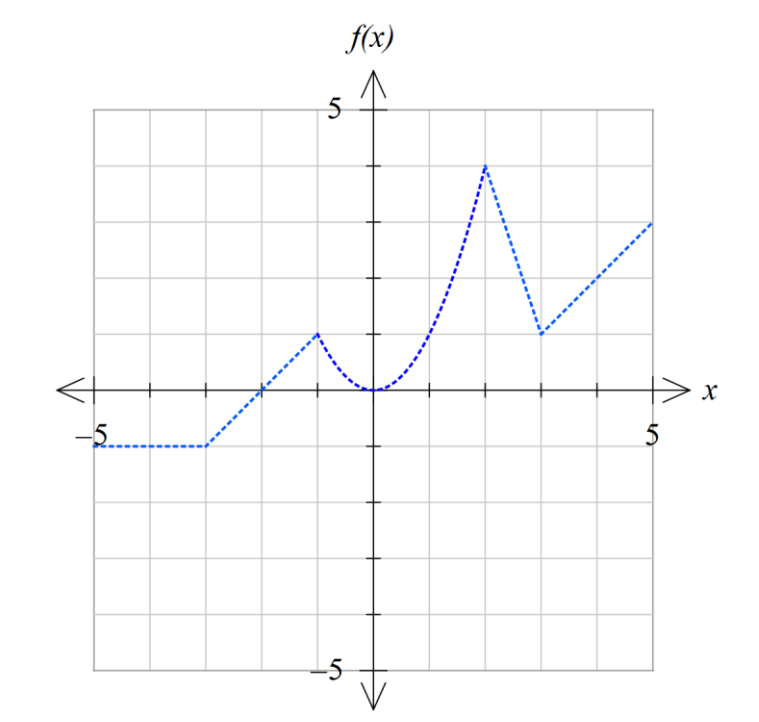
7.   (6 marks)

Consider the following function

(a)   Sketch $\dfrac{1}{f(x)}$ [2]
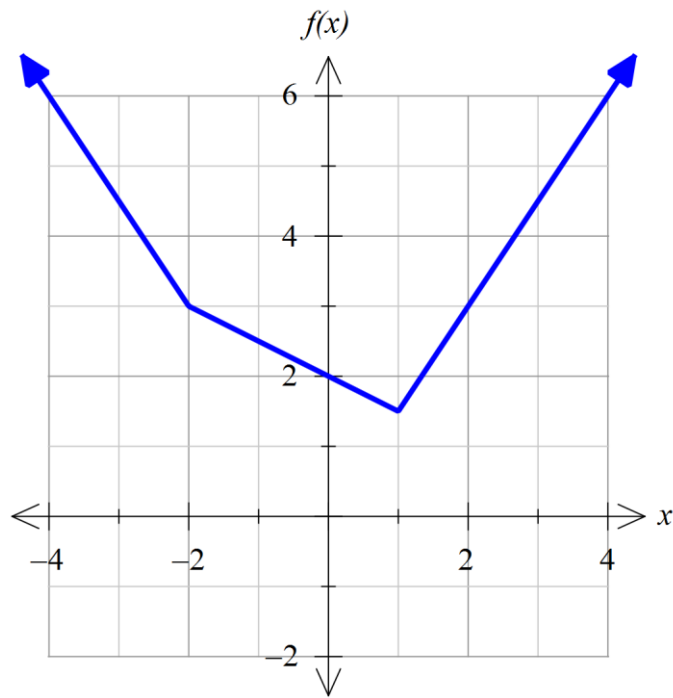
(c)   Sketch $\left|f\left(\left|x\right|\right)\right|$ [2]

(d)   Hence, or otherwise, solve $f(x)\left|f\left(\left|x\right|\right)\right|=1$ for $x\geq 0$ [2]

8.   (5 marks)

The graph of $f(x) = |x-1| + \left|\dfrac{x}{2}+1\right|$ is given below:

The solution to the equation $a|x+b|+c = |x-1| + \left|\dfrac{x}{2}+1\right|$ is $\{x: -1 \le x \le 1\}$.

(a)   Sketch a possible graph of $g(x) = a|x+b|+c$ on the axes above.   [2]

(b)   Determine the values of the real constants a , b and c .   [3]